

# Bayesian Estimation of the Survival Function

A. Lavanya<sup>1</sup>, T. Leo Alexander<sup>2</sup>

Research Scholar, Department of Statistics, Loyola College, Chennai-34, India<sup>1</sup>

Associate Professor, Department of Statistics, Loyola College, Chennai-34, India<sup>2</sup>

**ABSTRACT:** Weibull distribution is one of the most important lifetime distributions that is used for modeling and analysing data in clinical, life sciences and engineering. The main objective of this paper was to determine the best Bayesian Survival estimator for the Constant Shape Bi-Weibull distribution. The methods under consideration are the Bayesian estimator by using three loss functions, which are Linear Exponential (LINEX) loss, General Entropy loss and Squared Error loss function. Lindley approximation is used to obtain the Bayes estimators. Comparisons are made through simulation study to determine the performance of these methods. An illustrative example is also provided to explain the concepts.

**KEYWORDS:** Bayesian, MLE, Jeffreys Prior information, Extension of Jeffreys Prior information, Lindley approximation.

## I. INTRODUCTION

Weibull distribution is mostly used for failure time in both lifetime and reliability analysis. It was mostly very useful for modeling and analyzing life time data in the applied and engineering sciences [1]. In [2] stated that the advantage of Weibull analysis is its ability to provide accurate failure analysis and failure forecasts with extremely smaller samples. Previously Bayesian survival estimator for Weibull distribution with censored data was discussed in [3]. In this paper the main consideration is to determine the best Bayesian Survival estimator for the Constant Shape Bi-Weibull distribution by using three loss functions, which are Linear Exponential (LINEX) loss, General Entropy loss and Squared Error loss function. Lindley approximation is used to obtain the Bayes estimators.

We also studied Estimation of the Survival Function under the Constant Shape Bi-Weibull Failure Time Distribution Based on Three Loss Functions [4], we used only Extension of Jeffreys prior information but in this paper we have to use both methods, namely, Extension of Jeffreys prior information and Jeffrey's prior information for best Bayesian Survival Estimator.

This paper is organized as follows. In the Section 2, Bayesian estimation of Survival function under Jeffreys' Prior Information and Extension of Jeffreys' Prior Information with Three Loss functions is discussed. Section 3, provides simulation study carried out to investigate the performance of the Survival estimators. In Section 4, the proposed theory is validated by using simulated data. This is followed by results and finally conclusion is given in section 5.

## II. BAYESIAN ESTIMATION OF SURVIVAL FUNCTION

Let  $t_1, t_2, \dots, t_n$  be a random sample of size  $n$  with respect to the Constant Shape Bi-Weibull distribution, with  $\sigma$  and  $\beta$  as the parameters, where  $\sigma$  is the scale parameter and  $\beta$  is the shape parameter. The probability density function (*pdf*), cumulative distribution function (*cdf*) and survival function are given, respectively, as follows:

$$f(t; \sigma, \beta) = \frac{\beta}{\sigma} t^{\beta-1} e^{-\left(\frac{t^\beta}{\sigma}\right)}. \quad (1)$$

The Cumulative distribution function (CDF) is

$$F(t; \sigma, \beta) = 1 - e^{-\left(\frac{t^\beta}{\sigma}\right)}. \quad (2)$$

The Survival function is

$$S(t; \sigma, \beta) = e^{-\left(\frac{t^\beta}{\sigma}\right)}. \quad (3)$$

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

For analysing Failure time data Bayesian survival estimation approach has received a lot of attention. Prior Distribution and the Loss Function in Bayesian analysis the choice of prior is inevitable. The choice of a prior, most often does not depend on ones subjective knowledge and beliefs.

The likelihood function of the selected sample is given by

$$L(t_i, \sigma, \beta) = \prod_{i=1}^n \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left(\frac{t_i}{\sigma}\right)^\beta} \quad (4)$$

The log-likelihood function is

$$\ln L = n \ln \beta + (\beta - 1) \left[ \sum_{i=1}^m \ln t_i \right] - n \ln \sigma - \frac{1}{\sigma} \sum_{i=1}^n t_i^\beta \quad (5)$$

The Bayes estimators of the parameters are considered with different loss functions which are given below:

$$\text{Linex loss: } L(\hat{\theta} - \theta) \propto \exp(a(\hat{\theta} - \theta)) - a(\hat{\theta} - \theta) - 1 \quad (6)$$

$$\text{General entropy loss: } L(\hat{\theta} - \theta) \propto \left(\frac{\hat{\theta}}{\theta}\right)^k - k \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1 \quad (7)$$

and

$$\text{Squared Error loss: } L(\hat{\theta} - \theta) \propto (\hat{\theta} - \theta)^2 \quad (8)$$

## A. JEFFREY'S PRIOR INFORMATION:

Consider a likelihood function  $L(\theta)$ , with its Fisher Information  $I(\theta) = -E\left(\frac{\partial^2 \ln L}{\partial \theta^2}\right)$ . The Fisher Information measures the sensitivity of an estimator. Jeffreys suggested that  $\pi(\theta) \propto [I(\theta)]^{\frac{1}{2}}$  be considered as a prior for the likelihood function  $L(\theta)$ .

The Jeffreys prior is justified on the grounds of its invariance under parameterization according to Sinha [5]. Under the two-parameter Weibull distribution the non-informative (vague) prior according to Sinha and Sloan [6] is given as

$$v(\sigma, \beta) \propto \left[\frac{1}{\sigma\beta}\right]$$

Let the likelihood equation which is  $L(t_i | \sigma, \beta)$  being the same as (6). The joint posterior of the parameters  $\sigma$  and  $\beta$  is given by

$$\pi(\sigma, \beta | t) \propto L(t | \sigma, \beta) v(\sigma, \beta) \quad (9)$$

The marginal distribution function is the double integral of equation (9). Therefore, the posterior probability density function of  $\sigma$  and  $\beta$  given the data  $(t_1, t_2, \dots, t_n)$  is obtained by dividing the joint posterior density function over the marginal distribution function as

$$\pi^*(\sigma, \beta | t) = \frac{L(t | \sigma, \beta) v(\sigma, \beta)}{\iint L(t | \sigma, \beta) v(\sigma, \beta) d\sigma d\beta} \quad (10)$$

## B. EXTENSION OF JEFFREY'S PRIOR INFORMATION

We propose a Extension of Jeffreys Prior information such that,

$$u(\theta) \propto \left[\frac{1}{\theta}\right]^{2c}$$

This is an Extension of Jeffreys Prior information, when  $a=1$ , we have the standard Jeffreys Prior information and undefined when  $a = 0$ . Since our knowledge on the parameters is limited as a result of which a Jeffreys Prior

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

information approach is employed on both parameters. It is important that one ensures the prior does not significantly influence the final result. If our limited or lack of knowledge influences the results, then one may end-up giving wrong interpretations, which could affect whatever we seek to address. It is as a result of this that the Extension of Jeffreys Prior information is considered.

We have,

$$u(\sigma, \beta) \propto \left[ \frac{1}{\sigma\beta} \right]^{2c}$$

The likelihood function from equation (4) is

$$L(t_i | \sigma, \beta) = \prod_{i=1}^n \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left(\frac{t_i\beta}{\sigma}\right)}$$

According to Bayes theorem, the joint posterior distribution of the parameters  $\sigma$  and  $\beta$  is

$$\pi^*(\sigma, \beta | t) \propto L(t | \sigma, \beta) u(\sigma, \beta),$$

where

$$L(t_i | \sigma, \beta) = \frac{k}{(\sigma\beta)^{2c}} \prod_{i=1}^n \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left(\frac{t_i\beta}{\sigma}\right)}$$

and the marginal distribution is

$$\iint \frac{k}{(\sigma\beta)^{2c}} \prod_{i=1}^n \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left(\frac{t_i\beta}{\sigma}\right)} d\sigma d\beta,$$

where  $k$  is the normalizing constant that makes  $\pi^*$  a proper pdf. The posterior density function is obtained by using equation (10).

In recent times for analyzing Failure Time data Bayesian Estimation approach has received a lot of attention, which has mostly used to compare the traditional methods.

It may be note that by taking into account our prior information which is essential, the Bayes estimator shall be considered with respect to three loss functions which are also indisputable in Bayesian estimation.

## REMARK:

Here we consider two Asymmetric Loss Functions namely Linear Exponential (LINEX) Loss Function and General Entropy Loss Function. Also the Symmetric Loss Function namely Squared Error Loss Function considered in order to estimate Survival Function.

## C. LINEAR EXPONENTIAL LOSS FUNCTION (LINEX):

The LINEX Loss Function is under the assumption that the minimal loss occurs at  $\hat{\theta} = \theta$  and is expressed as

$$L(\hat{\theta} - \theta) \propto \exp(a(\hat{\theta} - \theta)) - a(\hat{\theta} - \theta) - 1,$$

where  $\hat{\theta}$  is an estimation of  $\theta$  and  $a \neq 0$ . The sign and magnitude of the shape parameter 'a' represents the direction and degree of symmetry, respectively. There is overestimation if  $a > 0$  and underestimation if  $a < 0$  but when  $a \cong 0$ , the LINEX Loss Function is approximately the Squared Error Loss Function. The posterior expectation of the LINEX Loss Function, according to [10], is

$$E_{\theta} L(\hat{\theta} - \theta) \propto \exp(a\hat{\theta}) E_{\theta}(\exp(-a\theta)) - a(\hat{\theta} - E_{\theta}(\theta)) - 1. \quad (11)$$

The Bayes Estimator of  $\theta$ , represented by  $\hat{\theta}_{BL}$  under LINEX Loss Function, is the value of  $\hat{\theta}$  which minimizes equation (11) and is given as

$$\hat{\theta}_{BL} = -\frac{1}{a} \ln E_{\theta}(\exp(-a\theta)).$$

Provided  $E_{\theta}(\exp(-a\theta))$  exists and is finite.

The posterior density function of the survival function under LINEX loss is given as

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

$$\hat{S}(t)_{BL} = E \left\{ \exp \left( -ae^{-\left(\frac{t_i^\beta}{\sigma}\right)} \right) \middle| t_i \right\} = \frac{\iint \exp \left( -ae^{-\left(\frac{t_i^\beta}{\sigma}\right)} \right) \pi^*(\sigma, \beta) d\sigma d\beta}{\iint \pi^*(\sigma, \beta) d\sigma d\beta} \quad (12)$$

From (12), it can be observed that ratio of integrals which cannot be solved analytically and so we employ Lindley's approximation procedure to estimate the parameters. Lindley considered an approximation for the ratio of integrals for evaluating the posterior expectation of an arbitrary function  $\hat{u}(\theta)$  as

$$E[u(\theta)|x] = \frac{\int u(\theta)v(\theta)[L(\theta)]d\theta}{\int v(\theta)[L(\theta)]d\theta}.$$

According to [6], Lindley's expansion can be approximated asymptotically by

$$\hat{\theta} = u + \frac{1}{2}[u_{11}\delta_{11} + u_{22}\delta_{22}] + u_1\rho_1\delta_{11} + u_2\rho_2\delta_{22} + \frac{1}{2}[L_{30}u_1\delta_{11}^2 + L_{03}u_2\delta_{22}^2], \quad (13)$$

where  $L$  is the log-likelihood function in equation (5),

$$u = \exp \left( -ae^{-\left[\frac{t_i^\beta}{\sigma}\right]} \right),$$

$$q = e^{-\left[\frac{t_i^\beta}{\sigma}\right]},$$

$$u_1 = \frac{\partial u}{\partial \sigma} = auq \left( \frac{t_i^\beta}{\sigma^2} \right), \quad u_{11} = \frac{\partial^2 u}{\partial \sigma^2} = uq \left( \frac{(at_i^\beta)^2 q - a(t_i^\beta)^2}{\sigma^4} + \frac{2at_i^\beta}{\sigma^3} \right),$$

$$u_2 = \frac{\partial u}{\partial \beta} = \frac{auqt_i^\beta \ln t_i}{\sigma}, \quad u_{22} = \frac{\partial^2 u}{\partial \beta^2} = uq \left[ \frac{(\ln t_i)^2 at_i^\beta}{\sigma} + \left( \frac{a \ln t_i t_i^\beta}{\sigma} \right)^2 \right],$$

$$\delta_{11} = (-L_{20})^{-1}, \quad \delta_{22} = (-L_{02})^{-1}.$$

$$L_{02} = -\left(\frac{n}{\beta^2}\right) - \frac{1}{\sigma} \sum_{i=1}^n t_i^\beta (\ln t_i)^2,$$

$$L_{03} = 2\left(\frac{n}{\beta^3}\right) - \frac{1}{\sigma} \sum_{i=1}^n t_i^\beta (\ln t_i)^3,$$

$$L_{20} = \frac{n}{\sigma^2} - 2 \frac{\sum_{i=1}^n t_i^\beta}{\sigma^3}, \quad \text{and } L_{30} = -2 \frac{n}{\sigma^3} + 6 \frac{\sum_{i=1}^n t_i^\beta}{\sigma^4}.$$

For Extension of Jeffreys Prior information

$$\rho(\sigma, \beta) = -\ln(\sigma^{2c}) - \ln(\beta^{2c}), \quad \rho_1 = \frac{\partial \rho}{\partial \sigma} = -\frac{1}{\sigma^{2c}} \quad \text{and} \quad \rho_2 = \frac{\partial \rho}{\partial \beta} = -\frac{1}{\beta^{2c}}.$$

For Jeffreys Prior information

$$\rho(\sigma, \beta) = -\ln(\sigma) - \ln(\beta), \quad \rho_1 = \frac{\partial \rho}{\partial \sigma} = -\frac{1}{\sigma} \quad \text{and} \quad \rho_2 = \frac{\partial \rho}{\partial \beta} = -\frac{1}{\beta}.$$

Bayesian Estimation of Survival Function using LINEX Loss Function is given as  $S(\widehat{t}; \sigma, \beta)_{BL} = e^{-\left(\frac{t^\beta}{\hat{\sigma}_{BL}}\right)}$ .

## D. GENERAL ENTROPY LOSS FUNCTION:

Another useful Asymmetric Loss Function is the General Entropy (GE) Loss which is a generalization of the Entropy

Loss and is given as  $L(\hat{\theta} - \theta) \propto \left(\frac{\hat{\theta}}{\theta}\right)^k - k \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1$ .

The Bayes Estimator  $\hat{\theta}_{BG}$  of  $\theta$  under the General Entropy Loss is  $\hat{\theta}_{BG} = [E_\theta(\theta^{-k})]^{-\frac{1}{k}}$ , provided  $E_\theta(\theta^{-k})$  exists and is finite. The posterior density function of the Survival function under general entropy loss is given as

$$\hat{S}(t)_{BG} = E \left\{ \left( \exp \left[ \frac{-t_i^\beta}{\sigma} \right] \right)^{-k} \middle| t_i \right\} = \frac{\iint \left( \exp \left[ \frac{-t_i^\beta}{\sigma} \right] \right)^{-k} \pi^*(\sigma, \beta) d\sigma d\beta}{\iint \pi^*(\sigma, \beta) d\sigma d\beta}.$$

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

Applying the same Lindley approach here as in (13) with  $u_1, u_{11}$  and  $u_2, u_{22}$  are the first and second derivatives for  $\sigma$  and  $\beta$ , respectively, and are given as

$$u = \left( \exp \left[ \frac{-t_i^\beta}{\sigma} \right] \right)^{-k}, e = \left[ \frac{-t_i^\beta}{\sigma} \right],$$

$$u_1 = \frac{\partial u}{\partial \sigma} = \frac{k}{\sigma} u e, u_{11} = \frac{\partial^2 u}{\partial^2 \sigma} = u e^2 \left( \frac{k}{\sigma} \right)^2 - \frac{2kue}{\sigma^2},$$

$$u_2 = -kue(\ln t_i) \text{ and } u_{22} = \frac{\partial^2 u}{\partial^2 \beta} = -kue(\ln t_i)^2 + u(ke \ln t_i)^2.$$

Bayesian Estimation of Survival Function using General Entropy Loss Function is given as  $S(\widehat{t}; \sigma, \beta)_{BG} = e^{-\left(\frac{t^\beta_{BG}}{\hat{\sigma}_{BG}}\right)}$ .

### E. SYMMETRIC LOSS FUNCTION:

The Symmetric Loss Function is the Squared Error Loss is given by  $L(\hat{\theta} - \theta) \propto (\hat{\theta} - \theta)^2$ .

This Loss Function is symmetric in nature, that is, it gives equal weightage to both over and under estimation. In real life, we encounter many situations where overestimation may be more serious than underestimation or vice versa.

The most common loss function used for Bayesian estimation is the squared error (SE), also called quadratic loss. The square error loss denotes the punishment in using to  $\hat{\theta}$  estimate  $\theta$  and is given as  $E_\theta(t|\theta) = (\hat{\theta}(t) - \theta)^2$ , where the expectation is taken over the joint distribution of  $\theta$  and  $(t)$ .

The posterior density function of the Survival function under the Symmetric loss function are given as

$$\hat{S}(t)_{BS} = E \left\{ \exp \left[ \frac{-t_i^\beta}{\sigma} \right] | t_i \right\} = \frac{\iint \exp \left[ \frac{-t_i^\beta}{\sigma} \right] \pi^*(\sigma, \beta) d\sigma d\beta}{\iint \pi^*(\sigma, \beta) d\sigma d\beta}.$$

Applying the same Lindley approach here as in (13) with  $u_1, u_{11}$  and  $u_2, u_{22}$  are the first and second derivatives for  $\sigma$  and  $\beta$ , respectively, and are given as

$$u = \exp \left[ \frac{-t_i^\beta}{\sigma} \right], e = \left[ \frac{-t_i^\beta}{\sigma} \right],$$

$$u_1 = \frac{\partial u}{\partial \sigma} = \frac{-ue}{\sigma}, u_{11} = \frac{\partial^2 u}{\partial^2 \sigma} = u \left( \frac{e}{\sigma} \right)^2 - \frac{2ue}{\sigma^2},$$

$$u_2 = ue(\ln t_i) \text{ and } u_{22} = \frac{\partial^2 u}{\partial^2 \beta} = ue \ln t_i (e \ln t_i + 1).$$

Bayesian Estimation of Survival Function using Symmetric Loss Function is given as  $S(\widehat{t}; \sigma, \beta)_{BS} = e^{-\left(\frac{t^\beta_{BS}}{\hat{\sigma}_{BS}}\right)}$ .

### III. SIMULATION STUDY

Since it is difficult to compare the performance of the estimators theoretically and also to validate the data employed in this paper, we have performed extensive simulations to compare the estimators through Mean Squared Errors and Absolute Biases by employing different sample sizes with different parameter values. The Mean Squared Error and

Absolute Bias given as  $MSE = \frac{\sum_{r=1}^{5000} (\hat{\theta}^r - \theta)^2}{R-1}$  and  $Abs = \frac{\sum_{r=1}^{5000} |\hat{\theta}^r - \theta|}{R-1}$ .

In our Simulation study, we chose a sample size of  $n = 25, 50,$  and  $100$  to represent small, medium, and large dataset. The Survival function is estimated for Constant Shape Bi-Weibull distribution with Bayesian using Extension of Jeffreys' Prior method and Jeffreys' Prior method.

The values of the parameters chosen are  $\sigma = 0.5$  and  $1.5,$   $\beta = 0.8$  and  $1.2$ . The values of Jeffreys Extension are  $c = 0.4$  and  $1.4$ . The Loss parameters values are  $a = k = (\pm 0.6, \pm 1.6)$ . These were iterated ( $R$ ) 5000 times and the Survival function for each method was calculated. The results are presented below for the estimated Survival function and their corresponding Mean Squared Error and Absolute Bias values.

In Table 1 we present the Mean Square Error estimated values for the Survival function  $S(t)$  for Bayesian Estimation using extension of Jeffrey's prior information with the three loss functions.

## International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

**Table 1:** MSE Estimated Survival function using extension of Jeffrey's prior information

n	$\sigma$	c	$\beta$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
					a = k = 0.6		a = k = -0.6		a = k = 1.6		a = k = -1.6	
25	0.5	0.4	0.8	4.2007	0.8488	283.32	2.8479	3.0503	2.5868	3706865	66.008	4.5070
	0.5	1.4	0.8	4.2695	0.8889	222.49	2.9214	3.0062	2.7562	2108747	66.675	4.6025
	0.5	0.4	1.2	4.2584	0.8739	116.83	3.0369	2.8955	2.5562	301937.8	70.796	4.8910
	0.5	1.4	1.2	4.4609	0.3570	6.4981	1.8970	1.6525	0.7064	162.5991	59.478	4.6074
	1.5	0.4	0.8	4.4838	1.0216	163.43	3.0882	3.3407	3.6395	418602	67.561	4.9069
	1.5	1.4	0.8	4.5255	1.0786	175.05	3.4672	3.2399	3.7418	869294.6	79.953	5.3834
	1.5	0.4	1.2	3.5733	0.7831	298.91	2.4931	2.7722	2.6609	2972000	57.149	3.9302
	1.5	1.4	1.2	3.5633	0.8028	162.10	2.1534	2.9026	2.8644	305725.5	39.212	3.2469
50	0.5	0.4	0.8	8.0489	1.7025	1415.261	4.9239	6.9123	5.7190	40735862	98.5131	7.2818
	0.5	1.4	0.8	7.6888	1.5836	1066.58	5.1215	6.0518	4.9388	5025113880	113.234	8.0862
	0.5	0.4	1.2	7.7564	1.6212	102.2422	5.8756	5.1781	4.6015	52547.17	142.192	9.5131
	0.5	1.4	1.2	6.4410	1.4276	56.94168	4.9212	4.0470	4.1203	20207.87	111.920	7.7936
	1.5	0.4	0.8	6.8489	1.4921	480.746	4.5774	5.2423	5.0041	10373194	98.7605	7.2698
	1.5	1.4	0.8	7.1812	1.5218	132.2614	4.9586	4.9977	4.6629	213972.6	107.548	8.0903
	1.5	0.4	1.2	10.146	2.1550	1047.962	6.6714	8.2033	6.9864	23857352	141.605	10.383
	1.5	1.4	1.2	7.6612	1.6268	893.232	5.1654	5.8401	5.1644	58425573	112.276	8.2034
100	0.5	0.4	0.8	15.9599	3.2816	771.6215	10.539	12.5423	10.2787	2522011	232.4689	16.5723
	0.5	1.4	0.8	16.2185	3.3791	991.9134	10.476	12.9333	10.7703	5437513	221.4281	16.2057
	0.5	0.4	1.2	15.8053	3.1785	350.2333	11.074	11.0590	9.1398	633570.3	254.8635	18.2381
	0.5	1.4	1.2	16.1742	3.3046	553.6934	11.677	11.4502	9.5553	1869227	275.4141	19.0687
	1.5	0.4	0.8	12.8229	2.6007	241.7176	9.3481	8.59889	7.41679	371204.2	221.3958	15.7038
	1.5	1.4	0.8	17.9673	3.6025	573.0519	13.268	12.1827	10.1714	5510193	324.8459	22.2644
	1.5	0.4	1.2	16.2717	3.4014	4220.786	10.771	12.7988	10.7625	2634290550	232.7989	17.0002
	1.5	1.4	1.2	17.8387	3.7973	5046.634	11.461	15.1484	12.4771	833552987	237.7222	17.4871

BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function

From Table 1 it is observed that Bayes estimation with LINEX loss function provides the smallest MSE values in most cases especially the loss parameter value is 0.6. Also when sample size increases Bayes estimation under all loss functions have increases in MSE values.

In Table 2 we present the Mean Square Error estimated values for the Survival function S(t) for Bayesian Estimation using Jeffrey's prior information with the three loss functions.

**Table 2:** MSE Estimated Survival function using Jeffrey's prior information

n	$\Sigma$	$\beta$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
				a = k = 0.6		a = k = -0.6		a = k = 1.6		a = k = -1.6	
25	0.5	0.8	4.2040	0.8507	280.0695	2.8515	3.0476	2.5947	3619255	66.0459	4.5117
	0.5	1.2	4.2568	0.8765	114.9991	3.0417	2.8891	2.5669	290793.4	70.83233	4.8898
	1.5	0.8	4.4858	1.0205	165.4165	3.0869	3.3464	3.6342	427904.6	67.55801	4.9069
	1.5	1.2	3.5713	0.7817	300.787	2.4899	2.7730	2.6553	3000772	57.09578	3.9282
50	0.5	0.8	8.0525	1.7033	1409.696	8.0525	12.7030	5.7230	40365333	98.5172	7.2876
	0.5	1.2	7.7516	1.6254	101.1973	5.8853	5.1676	4.6177	51202.52	142.3266	9.5098
	1.5	0.8	6.8509	1.4914	484.4755	4.5769	5.2472	5.0007	10520720	98.7718	7.2699
	1.5	1.2	10.145	2.1541	1050.526	6.6699	8.2049	6.9820	23973017	141.5955	10.380
100	0.5	0.8	15.9607	3.2827	769.9133	10.5415	12.5396	10.2836	2507838	232.4938	16.5756
	0.5	1.2	15.7987	3.1809	348.5335	11.0784	11.0492	9.15006	625481.1	254.8652	18.2355
	1.5	0.8	12.8287	2.5993	242.7456	9.34571	8.60601	7.41109	374966.6	221.3864	15.7075
	1.5	1.2	16.2714	3.4005	4226.34	10.7698	12.8002	10.7585	2640829400	232.7852	16.9983

BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function

From Table 2 it is observed that Bayes estimation with LINEX loss function provides the smallest MSE values in most cases especially the loss parameter value is 0.6. Also when sample size increases Bayes estimation under all loss functions have increases in MSE values.

## International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

In Table 3 we present the Absolute Bias estimated values for the Survival function  $S(t)$  for the Bayesian Estimation using extension of Jeffrey's prior information with the three loss functions.

**Table 3:** Absolute Bias Estimated Survival function using extension of Jeffrey's prior information

$n$	$\sigma$	$C$	$\beta$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$		$\hat{S}(t)_{BG}$		$\hat{S}(t)_{BL}$		$\hat{S}(t)_{BG}$	
					a = k = 0.6		a = k = -0.6		a = k = 1.6		a = k = -1.6	
25	0.5	0.4	0.8	8.3771	3.7574	42.205	6.8163	7.0583	6.5095	2769.876	32.0473	8.5138
	0.5	1.4	0.8	8.4594	3.8508	38.053	6.9022	7.0031	6.7323	2178.599	32.1462	8.6570
	0.5	0.4	1.2	8.4150	3.8094	30.902	7.0578	6.8537	6.4476	936.924	33.4272	8.9189
	0.5	1.4	1.2	8.5101	2.3168	9.4690	5.5362	5.1109	3.0914	43.69849	31.4336	8.7222
	1.5	0.4	0.8	8.5690	4.0622	38.589	7.1401	7.2523	7.3924	1333.851	32.9919	8.9839
	1.5	1.4	0.8	8.3421	4.0545	35.454	7.5050	6.7953	7.1247	1578.155	36.3294	9.3166
	1.5	0.4	1.2	7.5891	3.5333	44.730	6.2690	6.4750	6.2992	2804.021	28.9329	7.8128
	1.5	1.4	1.2	7.6448	3.6233	41.675	5.9237	6.8530	6.7746	1290.754	25.0417	7.2151
50	0.5	0.4	0.8	16.2965	7.4762	138.144	12.7015	14.9500	13.5430	14238.09	55.8869	15.2814
	0.5	1.4	0.8	15.9800	7.2522	226.35	12.9078	13.9637	12.6718	300582.5	59.1368	16.0463
	0.5	0.4	1.2	15.9623	7.2751	44.618	13.9731	12.7727	11.9386	674.2519	68.0087	17.7564
	0.5	1.4	1.2	14.6302	6.8852	33.861	12.7955	11.4131	11.5010	378.8708	60.1096	16.0930
	1.5	0.4	0.8	14.9303	6.9205	71.873	12.2084	12.7571	12.2722	4760.676	55.5827	15.3249
	1.5	1.4	0.8	15.3583	7.0630	49.657	12.7042	12.7412	12.2030	981.3793	58.2463	16.1151
	1.5	0.4	1.2	18.2258	8.3666	127.71	14.7802	16.2118	14.8304	10179.21	67.4726	18.3025
	1.5	1.4	1.2	15.8075	7.2172	87.174	13.0657	13.4394	12.4588	10501	60.4905	16.4504
100	0.5	0.4	0.8	32.5273	14.7811	169.67	26.1317	28.7596	25.9441	5585.59	118.9032	32.1750
	0.5	1.4	0.8	32.4537	14.9191	191.51	25.6315	29.3227	26.7700	7823.17	113.7779	31.1111
	0.5	0.4	1.2	32.3324	14.4185	107.82	27.1531	26.6175	23.9232	2377.758	129.3271	34.7457
	0.5	1.4	1.2	32.6653	14.6922	123.22	27.8716	26.8611	24.3337	3939.297	134.0359	35.5271
	1.5	0.4	0.8	29.1602	13.1049	85.866	24.7767	23.5898	21.6657	1720.041	118.1209	31.9101
	1.5	1.4	0.8	34.3904	15.3049	117.97	29.6346	27.8819	25.0799	4734.798	145.5364	38.2679
	1.5	0.4	1.2	32.8574	15.0540	230.341	26.4771	29.0612	26.5405	26.5405	119.6786	32.7033
	1.5	1.4	1.2	34.0778	15.8355	343.012	26.9402	31.7276	28.7537	64707.08	118.7613	32.4007

BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function

From Table 3 it is observed that Bayes estimation with LINEX loss function provides the smallest Absolute Bias values in most cases. As the sample size increases Absolute values of the Bayes estimation under all loss functions increases. In Table 4 we present the Absolute Bias estimated values for the Survival function  $S(t)$  for the Bayesian Estimation using Jeffrey's prior information with the three loss functions.

**Table 4:** Absolute Bias Estimated Survival function using Jeffrey's prior information

$n$	$\Sigma$	$\beta$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$		$\hat{S}(t)_{BG}$		$\hat{S}(t)_{BL}$		$\hat{S}(t)_{BG}$	
				a = k = 0.6		a = k = -0.6		a = k = 1.6		a = k = -1.6	
25	0.5	0.8	8.3815	3.7619	41.999	6.8206	7.0550	6.5201	2742.315	32.0535	8.5206
	0.5	1.2	8.4140	3.8157	30.700	7.0640	6.8463	6.4624	922.4347	33.43658	8.9204
	1.5	0.8	8.5709	4.0598	38.792	7.1383	7.2585	7.3862	1346.016	32.9926	8.9829
	1.5	1.2	7.5852	3.5293	44.849	6.2641	6.4754	6.2904	2815.518	28.9180	7.8079
50	0.5	0.8	16.3005	7.4783	137.897	12.7030	14.9492	13.5484	14182.55	55.8865	15.2890
	0.5	1.2	15.9568	7.2864	44.4053	13.9866	12.7591	11.9630	666.7129	68.0421	17.7560
	1.5	0.8	14.9320	6.9184	72.0825	12.2072	12.7631	12.2666	4788.059	55.5876	15.3236
	1.5	1.2	18.2252	8.3646	127.8573	14.7785	16.2135	14.8252	10199.89	67.4715	18.2997
100	0.5	0.8	32.5287	14.7837	169.4987	26.1340	28.7564	25.9510	5571.953	118.9057	32.1809
	0.5	1.2	32.3253	14.4246	107.6088	27.1578	26.6051	23.9385	2365.377	129.3244	34.7445
	1.5	0.8	29.1665	13.1010	86.0094	24.7737	23.5998	21.6562	1726.829	118.1228	31.9121
	1.5	1.2	32.8567	15.0520	230.4652	26.4755	29.0628	26.5405	70497.09	119.6783	32.7003

BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 9, September 2017

From Table 4 it is observed that Bayes estimation with LINEX loss function provides the smallest Absolute Bias values in most cases. As the sample size increases Absolute values of the Bayes estimation under all loss functions increases.

## IV. ILLUSTRATION

The real data set is about a clinical Trial in the Treatment of Carcinoma of the Oropharynx (PHARYNX) Data extracted from [7]. The data file gives the data for a part of a large clinical trial carried out by the Radiation Therapy Oncology Group in the United States. This data consists of a total of 195 respondents of which 53 are alive and 142 are dead. Here we considered Survival time in days from day of diagnosis is the most factor.

Table 5 depicts the Standard Error values for Estimated Survival function S(t) using PHARYNX Data. So the Survival function is estimated for Constant Shape Bi-Weibull distribution with Bayesian using Extension of Jeffreys' Prior method and Jeffreys' Prior method.

**Table 5:** Standard Error values for Estimated Survival function S(t) using PHARYNX Data

Estimates		$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
		a = k = 0.6		a = k = -0.6		a=k=1.6		a=k=-1.6		
Extension of Jeffreys	c=0.4	2.2e-89	0.0000	2.9e+54	0.0000	2.6e-54	0.00	1.7e+147	0.0000	5.5e-142
	c=1.4	1.7e-87	0.0000	1.0e+57	0.0000	1.6e-52	0.00	6.8e+149	0.0000	1.7e+147
Jeffreys		2.2e-89	0.0000	3.0e+54	0.0000	2.6e-54	0.00	1.7e+147	0.0000	5.4e-142

From Table 5, we observe that, Bayesian estimator under LINEX loss function has the smallest values for Survival function S(t). So that the Bayes estimators of Survival function S(t) using Extension of Jeffreys' Prior method and Jeffreys' Prior method under LINEX loss function is best estimation method for Constant Shape Bi-Weibull Distribution using PHARYNX Data.

## V. CONCLUSION

In this paper, we observed that the problem of Bayesian estimation of Survival function for the Constant Shape Bi-Weibull distribution, under Asymmetric and Symmetric loss functions. Bayes estimators were obtained using Lindley approximation method. A Simulation study was conducted to examine and compare the performance of the estimators for different sample sizes with different values for the extension of Jeffreys' prior and the loss functions. From the results, we observe that in most cases, Bayesian estimator under LINEX loss function has the smallest Mean Squared Error values and minimum Bias for Survival function S(t) in most cases especially compared to the loss parameter values are 0.6 and 1.6, for both values of the extension of Jeffreys' prior information. As the sample size increases the Mean Squared Error and the Absolute Bias for Bayes estimator under all the loss functions increases correspondingly.

## REFERENCES

- [1] Lawless, J. F., "Statistical Models and Methods for Lifetime Data", John Wiley & Sons, New York, NY, USA, 1982.
- [2] Abernethy, R.B., "The New Weibull Handbook", 5th edition, 2006.
- [3] Al Omari, M.A., and Ibrahim, N.A., "Bayesian survival estimation for Weibull distribution with censored data", Journal of Applied Sciences, 11(2), 393-396, 2011.
- [4] Lavanya, A., and Leo Alexander, T., "Estimation of The Survival Function under The Constant Shape Bi-Weibull Failure Time Distribution Based on Three Loss Functions", International Journal of advanced Research, 4(9), 1225-1234, 2016.
- [5] Sinha, S. k., and Sloan, J. A., "Bayes Estimation of the Parameters and Reliability Function of the 3-Parameter Weibull Distribution", IEEE Transactions on Reliability, 37, 364-369, 1988.
- [6] Sinha, S. K., "Bayes Estimation of the Reliability Function and Hazard Rate of a Weibull Failure Time Distribution", Trabajos De Estadistica, 1, 47-56, 1986.
- [7] [www.umass.edu/statdata/statdata/data/pharynx.txt](http://www.umass.edu/statdata/statdata/data/pharynx.txt)